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Logistics Network Design, Shipment Consolidation in a Distribution Centre – Warehouses – Customers Integrated Inventory Model

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ABSTRACT

This paper presents a study on multi level customers from three integrated network design and management of inventory to distribute from the various centres. The distribution process is performed through the warehouses situated in different location. Then the assignment will be shipped through the agent of the distribution centres. The main objective of this inventory model is to reduce the transportation and handling costs in the logistics network. This network design to establish the proper shipment model for clear the inventory and to provide an equivalent linear formulation. To prove effectiveness of the model, the strategy for handling the inventory is also proposed to in this study. The results also suggested that the distribution centres with high holding capacities should be arranged and scheduled to deliver their shipments.

Key words: Shipment Consolidation; Logistics Network Design; Inventory Management; Multiple Distribution Centres; Integrated Inventory Model.

1. Introduction

In today's competitive environment, companies have outsourced non-value-adding processes and reduced their business activity to core competencies. In many industries, outsourcing ratios have reached 60% or more (e.g. Muller, 2009), which suggests that managing the supplier base can have a more impact on the cost position of the company. More studies considered the supplier relationship management as an important measure to develop the competitive position of the company (e.g. Dyer, 1996; Saunders, 1997). Production planners should include upstream and downstream companies in their decisions to synchronize material flows and to reduce inventory levels with value creation now taking place in difficult production networks instead of individual companies. If the industries looking after the individual companies, the transportation and handling costs and warehousing cost and so on. Starting with Goyal (1976), a stream of research has emerged in recent years that focuses on the coordination of material movements of semi-finished and finished products. Reducing the total cost of producing and distribution a product in the supply chain increases its competitiveness and produces a cooperation gain that can be delivered among the companies involved.

From the study of the literature reveals that models dealing with the coordination of inventory distribution centres have mainly focused on the sales view of the supply chain, but also taken into consideration the costs that reduce the transportation and handling. In a recent review of joint economic lot size model, Glock (2012) pointed only 7 out of 155 papers that considered more than a single supplier, while the other works focused on 1:1 or 1:n relationships. It is obvious that the studying delivery process from a single manufacturer to its customers can help to reduce transportation and inventory carrying costs, and that this may help to improve the cost positions of all companies involved.

However, it is required to study that connection of the manufacturer to its suppliers also need to be studied, and the problem of handling delivery time and quantities in a n:1 = scenario is not necessarily the same companies meet when coordinating deliveries from a producers to multiple distribution centres and to the multiple customers.

The previous research has shown that the timing and size of deliveries from the main distribution centres to the buyers are critical for the cost position of the supply chain (e.g. Glock, 2012 & Goyal, 2009). The predicament supply chain distributors have to face is that frequent distribution from the suppliers to the buyer reduce the costs related the transaction. Thus, if the cost of delivering products is high, it means directly the affects the costs earned from the distribution centres. Besides transportation cost, the consolidation of shipments also has a positive impact on the environment (Olku, 2012)

One characteristic that has been studied here is that in many practical situations. It is not compulsory to establish a direct connection between each supplier and the buyer. Hence, instead, if the suppliers are located in closer proximity to each other than to the buyer, it can be beneficial to

consolidate deliveries from manufacturers to the distribution centres. In such a case the buyer receives fewer shipments which results low transportation and handling costs, but increases inventory carrying cost. It is obvious that there is positive in the net benefit of the consolidation shipments.

The aim of this study is to investigate shipment consolidation in multi-distribution centres-multi level buyers integrated inventory model. The next part of this paper is designed as follows. The following two sections give an overview of related works and outline the assumptions and definitions that will be used in the remaining parts of this paper. Sec 4 develops a model of shipment consolidation, and Section 5 contains conclusion of this study and provision o suggestions and for future research.

2. Literature Review:

From the works of Goyal (1976) and Banerjee (1986), the cycle of the supply chain production, consumption and delivery cycles are concentrated structure to minimize the total cost . One stream of research for example, studied the impact of alternative shipment policies on the total cost of the supply chain and showed that cost can be reduced in many cases if production quantities are delivered in multiple delivers to the buyer(e.g., Chatterjee & Ravi, 1991); Goyal & Nebebe, 2000; Hill. 1997), Hill (1999) and Hill and Omar (2006) expressed that the optimal policy in a single-vendor- single-buyer model is to ship batches that increase in size by a fixed factor, followed by equal-size shipments. To the best of our knowledge, the first attempt to model and solve a problem of this class is given Candas and kutanoglu(2007) Examples for multi-state integrated inventory models are the works of Banerjee and Kim (1995), Khouja(2003), Abdul-jalbar, Gutierrez, and Sicilia(2007), Chung, Wee, and Yang(2008) and Kim and Glock (2013), who added raw material suppliers and distributors in their models.

In the integrated inventory models, the focus was given on single actor on each stage and studied scenarios where a single vendor has to deliver a product to multiple buyers. Several authors implemented a basic cycle time for the buyers in this case to avoid discrete and unequally spaced depletion form the vendor's inventory, which may result in shortages (see.e.g., Bannerjee & Burton, 1994; Chan & Kingsman, 2007; Siajadi, Ibrahim, & Lochert, 2006). Other authors focused on the consequences of vendor-managed inventory or consignment stock on the total cost of the system and showed that the total cost can be reduced if the vendor manages the warehouses at the buyers (e.g., Chen, Lin & Chen 2010: Darwish & Odah, 2010: Zavanella & Zanoni, 2009)

Consideration of multiple suppliers in an integrated inventory model is scarce. Ben-Daya and Al-Nassar (2008) identified one of the first models in this area and studied the case where multiple suppliers deliver to multiple manufacturers, who in turn supply to multiple buyers. The authors considered the case where each stage receives products from a single upstream stage only, wherefore problems that result from overlapping delivery cycles are not considered in this model. A related model is the one of the Jaber and Goyal (2008), who assumed that the suppliers deliver different product components to the buyers, who in turn assembles the components to a final product.

Chen and Sarker(2010) studied a buyer sourcing a product from multiple vendors and assumed that a single truck is used to collect the products from the vendors in a mile-run. A freight cost function, which considers both product weights and travel distances, was used to model freight costs and it was shown that the freight rates favour larger delivery quantities and fewer deliveries. The production lot size, however, was not affected by the cost of delivery. Kim and Goyal (2009) studied two different transportation strategies: in the case of lumpy deliveries, the vendors deliver simultaneously to the buyer, whereas in the case of phased deliveries, the vendors deliver alternately, such that a shipment reaches the buyer exactly when the shipment of the vendor who has delivered before has just been used up.

Glock(2012) studied a system with two vendors and a single buyer and compared the efficiency of six alternative delivery structures. He showed that permitting different lot sizes, shipment frequencies and production intervals at the suppliers reduces the total costs of the system. In

another paper, Glock (2011) studied a system with multiple heterogeneous suppliers and a single buyer and assumed that the buyer also faces a supplier selection problem. Deliveries of the suppliers were scheduled in such a way that suppliers deliver consecutively to the buyer and that deliveries do not overlap. Glock (2012) studied the case of multiple homogeneous suppliers delivering a product in equal sized shipments to a single buyer. He assumed that the production capacity of a single supplier does not suffice to satisfy demand at the buyer wherefore it is necessary that the delivery cycles of the suppliers overlap. This, in turn, leads to a steady increase in inventory over a cycle, which can be dampened by allocating large order quantities to suppliers that deliver first and smaller quantities to suppliers that deliver last in the sequence. Other works in this stream of research are the ones of Seliaman and Ahmad(2008), Sawik (2009), Leung(2010), Glock (2012) and Glock and Ries (2013).

Location and allocation decisions that take inventory decisions into account are known to be different from those that do not(Baumol & Wolfe, 1958). Nevertheless, inventory considerations even under deterministic assumptions present nonlinearities that may be quite challenging (Nozick & Turnquist, 2001). A number of integrated models are studied by Barahona and Jensen (1998), Eriebacher and Meller (2000), Teo, Ou, and Goh(2001) and Miranda and Garrido(2004, 2006) who introduce joint location-inventory models and exploit various heuristics or approximations to solve the resulting nonlinear optimization problems. Nonlinear location models with risk pooling are studied in Shen(2000, 2003) and Shu, Teo, and Shen (2005).

The purpose of this study is to analyse the shipment consolidation in an integrated inventory model. It assumes a buyer who replenishes its inventory from multiple vendors and who has the option to combine several of the vendors in vendors groups. The most related paper is one of Chen and Sarker (2010). However, this work differs from Chen and Sarker (2010) considered the special case of only a single supplier group. In addition, this paper assumes that the supplier delivers to the distribution centre for the individual or single buyers and also for the group of buyers simultaneously.

Finally, Chen and Sarker(2010) studied the case where the production rate of each supplier exceeds the demand rate of the buyer, whereas this work assumes that an individual supplier is not able to satisfy the entire demand at the buyer.

3.Model Development

Assumptions and Notations:

f_i -- Annual fixed location cost for warehouse i .

C_{ijk}^1 -- Unit transportation cost from distribution centre to warehouse i to destination j for part k.

c_{jkm}^2 -- Unit transportation cost from destination j to customer m for part k.

h_{ik} -- Unit holding cost for part k at warehouse i.

t_{jkm}^1 -- Transportation time from warehouse i to retailer j for part k

t_{jkm}^2 -- Transportation time from retailer j to customer m for part k

d_{jk} -- Random retailer j demand for part k with mean annual demand

d_{mk} --Random customer m demand for part k with mean annual demand Upper bound on stock level.

y_1 -- Binary location variable.

X_{ijk}^1 -- Long run fraction of retailer j demand for part k served from warehouse i.

X_{jkm}^2 -- Long-run fraction of customer m demand for part k served from retailer j .

S_{ik} --Stock level of part k at warehouse i.

w_k -- Time window for part k.

R_k -- Time window for part k from retailers j.

A -- Order costs per order for the retailers j.

C - Handling costs per shipment for the retailer j.

n – Number of available distribution centres i.

m_j - Number Of shipments of distribution centre(DC) i to the retailer in one order cycle, where

$$1 \leq i \leq d.$$

t_i - Normalised I^{st} shipment period from DC to the retailer with respect to T, where

$$t_i \in [0,1] \text{ \& } t = \{t_i \mid 1 \leq i \leq d \}.$$

α_i - Normalised shipment interval of DC i with respect to T, Where α_i is the fraction of T after which DC i delivers subsequant shipments.

δi_j - Binary variable that indicates whether vendor i is a member of group j, where$\sum_{j=1}^{d} \delta i_j = 1$ & $\sum_{i=1}^{n} \delta i_j \geq 1 \; \forall j$.

T - Order cycle length of the retailer

Q - Order Quantity per cycle, where Q = DT.

Definitions:

q_i -- Order Quantity allocated to distribution centre i ,

$$\text{where Q} = \sum_{i=1}^{n} q_i \text{ \& } q_i = Tp_i \sum_{j=1}^{d} \alpha_j m_j \; \delta i_j.$$

X_i -- Shipment quantity of DC group i with $X_i = \alpha_i T \sum_{i \in \Omega_i} pi$.

Ω_i -- Set of DC in consolidation group i, where $1 \leq i \leq d$.

Assumptions:

- The distribution centre (DC) has an unlimited capacity.
- Warehouse use a continuous review and (s-1, s) replenish policy with backordering.
- Retailer and Customers demand arrive only at a time according to an Independent Poisson Distribution.
- Parts are shipped directly to corresponding customers without consolidation or bundling.
- Customers are served on at 1st come 1st service basis regardless of their location
- Lateral transhipments are not allowed among warehouses.

The below figure shows inventory time plots for lot of size for alternative distribution groupings. After the retailer has completed his order, distribution centre starts production and delivers a batch after t_1 time units. The remaining 2, ... m_1 batches are delivered every α_1 time units after the first delivery. Since $D > \sum_{i \in \Omega_1} P_1$ in case that multiple groups are formed, the first batch of size x_1 is consumed at the retailer before the second batch from the same distribution group. Thus, the other distribution centre have to deliver at least one batch before the next shipment of the first distribution centre group arrives. However, due to $\sum_{i=1}^{n} P_1 \geq D$ the second batch of the first distribution centre arrives before the batches that were delivered before have been completely use up. This may result in an increase in inventory over time, as can be seen in the figure. Note that Glock (2012 a) observed a similar behaviour, and denoted stock that is on hand when shipment with ,... of distribution centre arrives as ' Cycle surplus'.
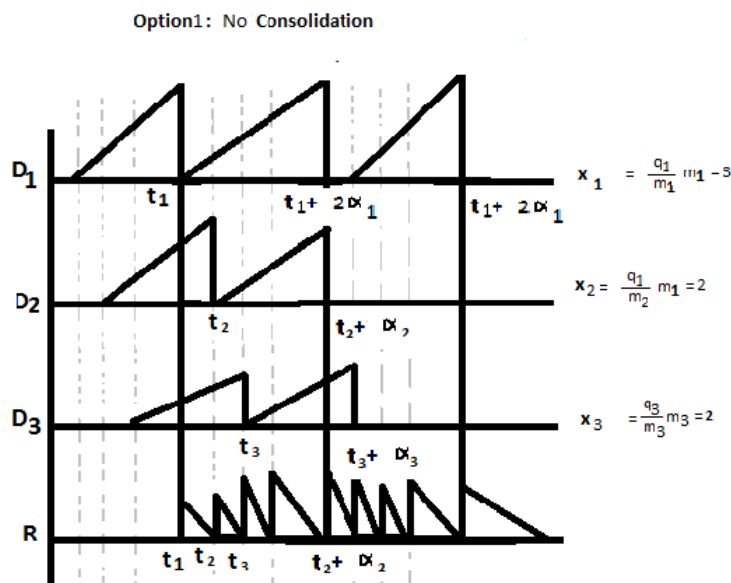
The average inventory at the retailer is computed by referring to the timings of the shipments from the Distribution Centres. A detailed derivation of the average inventory at the retailer, IR. It is obtained by dividing the time weighted inventory at the retailer by the cycle time T. The average inventory at the retailer is function of both the time of the 1st shipment of each Distribution Centres, t & the time between subsequent shipments of each Distribution Centres α ,
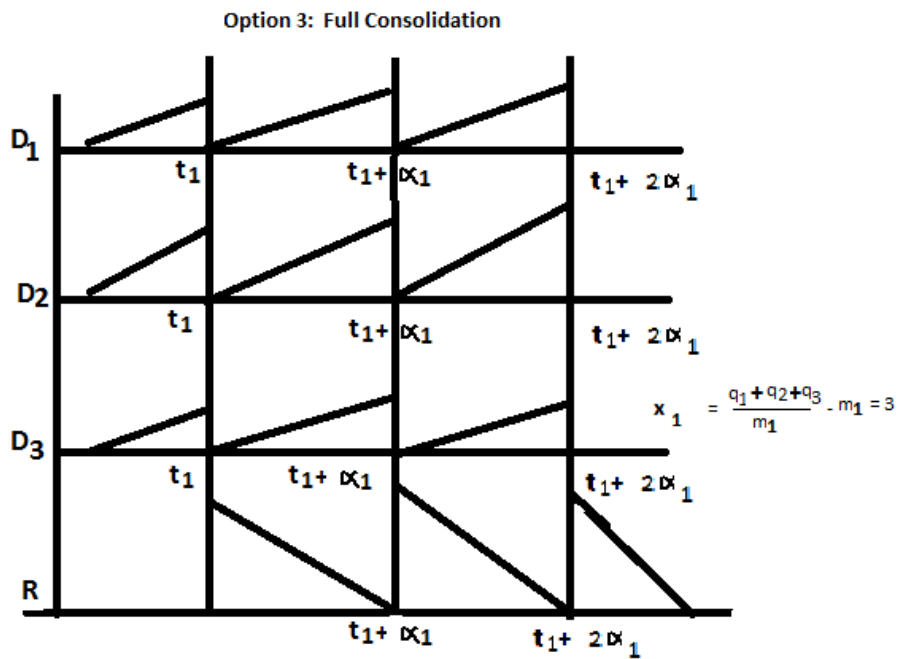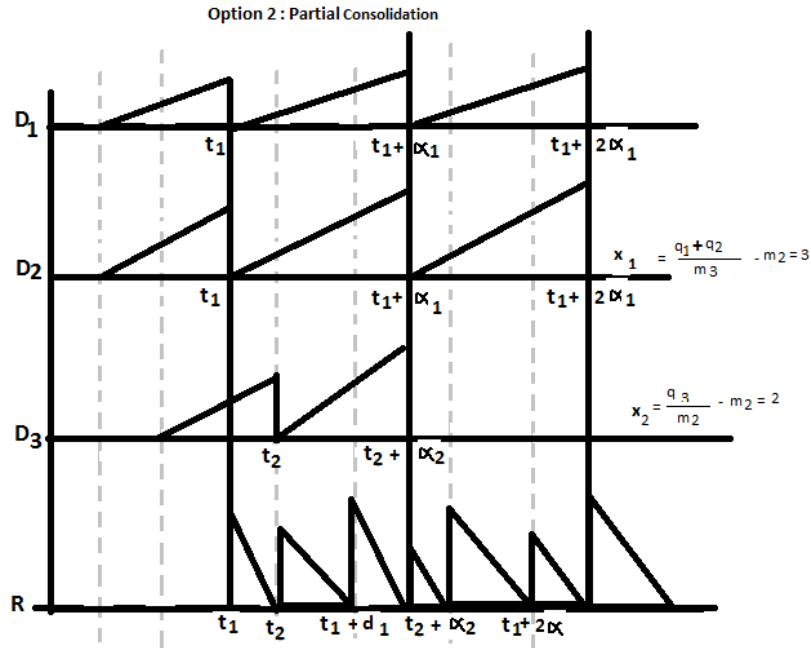
$$IR\,(m,T,\delta\,,t,\alpha\,) = \frac{T\omega J_r}{T} = \frac{DT}{2} - \sum_{j=1}^{d}\sum_{j=1}^{mj}(t_j + (i-1)\alpha_j) \text{-------(1)}.$$

Deriving the average inventory of Distribution Centres i in group j is straight forward & it is illustrated in (6).

$$IS_{ij}\,(m,T,\delta\,,\alpha\,) = m_j\left(\frac{(\alpha_j T)P_i(\alpha_j T)\delta_{ij}}{2T}\right) = m_j\,T\left(\frac{\alpha_j^2 P_j \delta_{ij}}{2}\right)$$

Where j = 1,2 ..... d , $x_j = \alpha_j T\left(\sum_{i \in \Omega_j} P_i\right)$

$$= \alpha_j T \sum_{i=1}^{n} P_i \delta_{ij} \text{ ....(2)}$$

Option1: No Consolidation

Option 2 : Partial Consolidation

Option 3: Full Consolidation

To avoid shortages at the retailer, several constraints for timings between consecutive shipments need to be considered specifically. We formulate two timing constraints for

$$t = (\, t_1, t_2 \ldots \ldots t_j \ldots . t_d) \ \& \ \alpha = \{\alpha_1, \alpha_2 \ldots \alpha_j \ldots \alpha_d\}.$$

$$t_j \ \leq \ \frac{\sum_{k=1}^{j-1} x_k}{DT} \ = \frac{\sum_{k=1}^{j-1} \sum_{i=1}^{n} \alpha_k P_i \delta_{ik}}{D}, \ where \ j = 2,3 \ldots . d \ldots \ldots \ldots (3)$$

$$\alpha_j \left(i_j \right) \leq \frac{\sum_{k \backslash j} \left(W_{j,k} \left((i_j) + 1 \right) \right) \alpha_k \sum_{i=1}^{n} P_i \delta_{ik} - t_j D}{(i_j - 1)D - i_j \sum_{i=1}^{n} P_i \delta_{ij}} - - - - - - - - - (4)$$

With $w_j, k\left(i_j\right) = \left(\frac{t_j - t_k + (i_j-1)\alpha_j}{\alpha_k}\right)$.

$\text{TRC}\,(T, m, \delta, t, \alpha, g) = \frac{\left(A + C\sum_{j=1}^{d} m_j + \sum_{i=1}^{n} S_i\right)}{T} + h^r IR(m, T, \delta, t, \alpha)$

$+$
$\sum_{j=1}^{d}\sum_{i=1}^{n} h_i^s IS_{ij}(m, T, \delta, \alpha) =$
 A+Cj=1dmj+i=1nSiT+T2 hrD− j=1d2tj+mj− 1αjmjαj i=1npiδij+j=1gmjαj2i=1nhispiδij

----------(5)

$T^* = \sqrt{\dfrac{2\left(A + c\sum_{i=1}^{d} m_j + \sum_{i=1}^{n} s_i\right)}{h^r\left(D - \sum_{j=1}^{d}\left(2t_j + (m_j - 1)\alpha_j\right)m_j\,\alpha_j \sum_{t=1}^{n} p_i\delta_{ij}\right)\right) + \sum_{j=1}^{d}\left(m_j\alpha_j^2 \sum_{i=1}^{n} h_i^s p_i\delta_{ij}\right)}}$

----------(6)

substituting (6) in (5),

$\text{TRC}\,(m, \delta, t, \alpha, g) =$

$$\sqrt{2\left(A + c\sum_{j=1}^{d} m_j + \sum_{i=1}^{n} S_i\right)\left(h^r\left(D - \sum_{j=1}^{d}\left(2t_j + (m_j - 1)\alpha_i\right)m_j\alpha_j\sum_{i=1}^{n} p_i\delta_{ij}\right) + \sum_{j=1}^{d}\left(m_j\alpha_j^2\sum_{i=1}^{n} h_i^s p_i\delta_{ij}\right)\right)}$$

---------------- (7)

Minimize TRC $(m, \delta, t, \alpha, g)$ in (7)

subject to $\left((i_j - 1)D - i_j\gamma_j\right)\alpha_j \le \sum_{k\backslash j}(t_j - t_k)\gamma_k - t_j D + (i_j - 1)\alpha_j \sum_{k\backslash j}\gamma_k$

where $i_j = 1,2 \dots m_j$, $j = 1,2,\dots\dots d$ & $\gamma_j = \sum_{i=1}^{n} p_i\delta_{ij}$ ....(8)

$m_j\,\alpha_j \le 1 \dots\dots\dots\dots..$ (9)

$$\sum_{i=1}^{n} m_j\alpha_j \sum_{i=1}^{n} p_i\delta_{ij} = D$$

where $\alpha_j > 0$, $t_j \ge 0$, $\delta_{ij} \in \{0,1\}$, $i = 1,2\dots n; j = 1,2\dots d \dots\dots\dots$ (10)

4. Linear Location Inventory Model:

Let $I_m = \{1,2,\dots m\}$, $I_n = \{1,2,\dots n\}$ & $I_p = \{1,2\dots p\}$ denote the index sets for potential warehouse locations, customers and parts respectively. An annual fixed cost f_i is acquired while creating an SPL system, for each open warehouse i, holding cost per unit h_{ik}, for part k at warehouse i, for part k at warehouse i and transportation cost per unit c_{ijk} for part k from warehouse i to customer j. The service time, which includes the transportation time from warehouse i to customer j for part k is denoted by τ_{ijk}. For part k, the lead time from the distribution centre to

warehouse i is t_{ik}. The demand of customer j for part k is a random variable d_{ik} with Poisson distribution and mean annual demand d_{ik}.

The three main sets of decision variables are applied. Variables $x_{ijk} \in \{0,1\}$ represents the long-run fraction of customer j's demand for part k fulfilled from warehouse i. Binary variable y_i takes value 1 if warehouse i is open, and 0 otherwise. Integer variable $S_{ik} \in IS_{max} = \{1,2,\dots\dots,s_{max}\}$ represents the base stock level of part k at warehouse i, where s_{max} is stocking capacity set a priori on the stock level of part k at a warehouse i.

We require that there exists at least one warehouse within the time window w_n of each retailer for each part & R_k of each customer for each part, ie.,

$$\left. \begin{array}{l} t_{jkm}^1 \;\le\; w_k \\ t_{jkm}^2 \;\le\; \overline{w_k} \end{array} \right\} \text{--------------(11)}$$

$$\forall \, j \in I_n \quad \forall \, k \in I_p \quad \exists \, i \in I_m \,.$$

$$\sum_{i \, \in \, I_m : t_{jkm}^1 \, \le w_t} X_{ijk}^1 \;=\; 1\,, \text{-- -- -- -- -- (12)} \; \forall \, j \in I_n \quad \forall \, k \in I_p$$

$$\sum_{i \, \in \, I_m : t_{jkm}^2 \, \le \overline{w_t}} X_{jkm}^2 \;=\; 1\,, \text{------------------------(13)} \; \forall \, i \in I_m \quad \forall \, k \in I_p$$

Since the demand of retailer j for part k is a random variable with Poisson Distribution & the demand of customer m for part k is a random variable with Poisson Distribution, it follows that demand for part k experienced at warehouse i during lead time is Poisson random variable with mean λ_{ik}

$$\lambda_{ik} \;=\; \left(t_{ik} + t_{jk}\right) \sum_{j \, \in \, I_n \,,\, t_{jkm}^1 \, \le w_t \,,\, t_{jkm}^2 \le \overline{w_t}} \left(d_{ik} + d_{jk}\right)\left(X_{ijk}^1 + X_{jkm}^2\right) \dots(14)$$

Given that the mean demand λ_{ik} and the stock level S_{ik} for part k at warehouse i, the achieved service levels for part k at each warehouse is defined as $\beta\,(\,\lambda_{ik}\,,\,S_{ik}\,) = \Pr(\widetilde{\lambda_{ik}} \;\le\; S_{ik} - 1)$.

Part warehouse specific service levels:

In this case, warehouse i is required to deliver the assigned demand of part k 100 α_{ik} % of time within the time window w_k. This is achieved when " the probability that stock probability during lead time is strictly larger than demand" is at least α_{ik}. In other words, the stock level s_{ik} of part k at an open warehouse i satisfies

$$s_{ik} \;\le\; s_{max}\, y_i \,, \forall \, i \in I_m \,, \forall \, k \in I_p \dots\dots\dots..(15)$$

$$Pr\left(\overline{\lambda_{ik}} \le s_{ik} - y_i\right) \ge \alpha_{ik} \,, \forall \, i \in I_m \,, \forall \, k \in I_p \dots\dots\dots\dots(16)$$

Non-linear model for the location inventory problem with part warehouse specific service levels, denoted by SM,

SM:

$$min \; \sum_{i \in I_m} f_i y_i + \sum_{(i,j,k) \in I_m * I_p * I_n} c_{ijk}\, d_{jk} \left(X_{ijk}^1 + X_{jkm}^2\right) + \sum_{(i,k) \in I_m I_n} h_{ik} S_{ik} \qquad \dots\dots..(17)$$

Subject to, $\sum_{i \, \in \, I_m : t_{jkm}^1 \, \le w_t} X_{ijk}^1 = 1 \,, \forall \, i \in I_m \quad \forall \, k \in I_p$

$$\sum_{i \, \in \, I_m : t_{jkm}^2 \, \le \overline{w_t}} X_{jkm}^2 = 1 \,, \; \forall \, j \in I_m \quad \forall \, k \in I_p$$

$0 \le x_{ijk} \le y_i , \forall \, i \in I_m , \forall \, j \in I_n , \; \forall \, k \in I_p \dots\dots\dots\dots(18)$

$s_{ik} \le s_{max}\, y_i, \quad \forall\, i \in I_m, \quad \forall\, k \in I_p \quad \dots\dots\text{(19)}$

$\beta\,(\lambda_{ik} \le s_{ik}) = P_r(\overline{\lambda_{ik}} \le s_{ik} - y_i) \ge \overline{\alpha_{ik}}, \forall\, i \in I_m, \quad \forall\, k \in I_p$
$\dots\dots\dots\text{(20)}$

$y_i \epsilon\{0,1\}, \forall\, i \in I_m \dots\dots\dots\text{(21)}$

$s_{ik} \in \{0,1,2\dots\dots\dots S_{max}\}, \forall\, i \in I_m, \quad \forall\, k \in I_p \dots\dots\text{(22)}$

The objective function (17) minimizes the total cost of opening warehouses, transportation and inventory holdings. Under Poisson demand the function $\beta\,(\lambda, S)\, in$ (20) is given by $\beta(\lambda, S) = e^{-\lambda}\, \sum_{r=0}^{s-1}\frac{\lambda^r}{r!}, \lambda \epsilon[0, \infty], S \in \{1,2\dots\dots\dots\}\dots\dots\text{(23)}$

where $\beta\ (0,S) = 1 \,\&\, \lim_{\lambda \to \infty}\beta(\lambda, s) = 0$. Finally the function is strictly monotonically decreasing with respect to $\lambda \epsilon[0, \infty]$.

Since $\frac{d}{d\lambda}\beta(\lambda, S) = -e^{-\lambda}\frac{\lambda^{S-1}}{(S-1)!} < 0 \dots\dots\text{(24)}$

For any $\alpha \epsilon (0,1), \beta(\lambda, S) = \alpha$ denoted by $\lambda(S, \alpha)$. We replace nonlinear constraints by the following linear equivalents.

$\sum_{s\epsilon IS_{max}} V_{iks} \le y_i, \forall\, i \in I_m, \quad \forall\, k \in I_p \dots\dots\dots\dots\text{(25)}$

$\lambda_{ik} \le \sum_{s\epsilon IS_{max}} \lambda_s(\alpha_{ik})\, V_{iks}, \forall\, i \in I_m, \quad \forall\, k \in I_p \dots\dots\dots\dots\text{(26)}$

where $Is_{max} = \{1,2\dots\dots s_{max}\} \,\&\, V_{iks}$ is a binary variables that takes value 1 if the base stock level part k at warehouse i is otherwise.

Replacing (19) & (20) by (25) & (26), we obtain an equivalent reformulation of the nonlinear mixed $IPP\ \textbf{SM}$ as a mixed IPP, denoted by $LMPW$.

$LMPW$:

$in \sum_{i\epsilon I_m} f_i y_i + \sum_{(i,j,k)\epsilon I_m * I_p * I_n} c_{ijk}\, d_{jk}\, x_{ijk} + \sum_{(i,k)\epsilon I_m * I_p * Is_{max}} Sh_{ik}\ V_{iks} \dots\dots\text{(27)}$

subject to
$\left.\begin{array}{l} \sum_{i\,\in\,I_m:t_{jkm}^1\ \le w_t}\ X_{ijk}^1 = 1, \forall\, i\epsilon I_m\quad \forall\, k\in I_p \\[6pt] \sum_{i\,\in\,I_m:t_{jkm}^2\ \le \overline{w_t}}X_{jkm}^2 = 1, \forall\, j\epsilon I_n\quad \forall\, k\in I_p \end{array}\right\}\dots\dots\dots\text{(28)}$

$0 \le x_{ijk} \le y_i, \forall\, i \in I_m, \forall\, j \in I_n, \quad \forall\, k \in I_p \dots\dots\dots\text{(29)}$

$\sum_{s\in Is_{max}} V_{iks} \le y_i, \forall\, i \in I_m, \quad \forall\, k \in I_p \dots\dots\dots\text{(30)}$

$\left.\begin{array}{l} t_{ik}\sum_{i\,\in\,I_m:t_{jkm}^1\ \le w_t} d_{ik}\, x_{ijk} \le \sum_{s\in Is_{max}}\lambda_s(\alpha_{ik})V_{iks}, \\[6pt] t_{jk}\sum_{j\,\in\,I_n:t_{jkm}^2\ \le w_t} d_{jk}\, x_{ijk} \le \sum_{s\in Is_{max}}\lambda_s(\alpha_{jk})V_{iks}, \end{array}\right\}, \forall\, i \in I_m, \forall\, j \in I_n, \quad \forall\, k \in I_p$

$\dots\dots\dots\dots\text{(31)}$

$y_i \epsilon \{0,1\}, \forall\, i \in I_m, \dots\dots\dots\text{(32)}$

$V_{iks} \in (0,1), \forall\, i \in I_m, \quad \forall\, k \in I_p, \forall S \in IS_{mas\dots\dots}) \text{(33)}$

In this model, the integer variable s_{ik} are removed and binary variables V_{iks} are introduced. For any open warehouse i & any part k, there must exist only one & s $s-t$ $V_{iks}=1$& for all cloed wareshouse. We must have $V_{iks}=0$. The stock level warehouse for part k is given by $s_{ik}=\sum_{s\in IS_{max}} SV_{iks}$. The values $\lambda f(x)$ are input data of LMPW calculated as the unique solutions of $\beta(\lambda, S)=\alpha$ with respect to λ for given α +s using a Newton method.

5.Conclusion

This paper studied the logistics network to design the supply chain towards the buyer through the distribution centres to warehouse and then to final buyer. The distribution centres are spreaded over to all the regions to easy access of the inventory with the intention to satisfy the supply of required or assigned products to the buyer in the supply chain. Whenever requires to supply, this network design reduces the risk of the supplier by the way of delivering the goods as soon as possible from the warehouse where the products were already distributed. To reduce costs and difficulties of inventory to the buyer, it is beneficial to organise the distribution centres in such a way that the buyer receives large shipments at the beginning and towards the end of the delivery cycle and small shipments in between. Further, it was shown that the distribution centres with high production capacities should be deliver in the middle of the delivery cycle, and that the distribution centres with low production capacities should be allocated to a delivery slot early or late in the distribution cycle.

However, this model has several limitations that should be addressed in the future research. First, we did not consider the cost for supplying the products to the distribution centres. Though the distribution process reduces the handling and transportation costs, the costs which spent from the place of producer to the place of distribution centres were not considered. It would also be interest to permit that vendors are part of multiple vendor groups. In this case, the system gains additional flexibility, since vendors with high production capacities can use the transportation equipment of other vendor groups without needing to increase the delivery frequency of the own vendor group.

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